Some properties of Definite Integrals

day 68

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx = \int_a^b (f \pm g)(x)\,dx$$

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

wherever b is

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

limit of integration "start here"

limit of integration "stop here"

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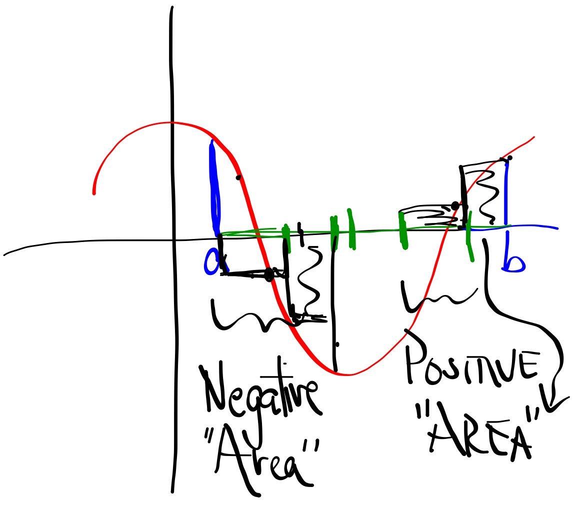
Definite Integrals and the IDEA of net area

Area under +-val. f^n between a & b $= \int_a^b f(x)\,dx = \lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \Delta x_k f(x_k^*)$



Def^n

Net Area is definite integral.

clumps below x-axis contribute negative area, clumps above x-axis contribute positive area.

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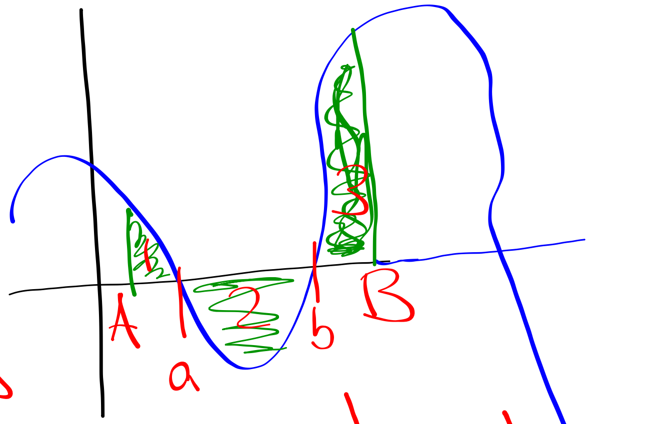
Two implications

1) Find area of



Implication:

$Area_1 + Area_2 + Area_3$

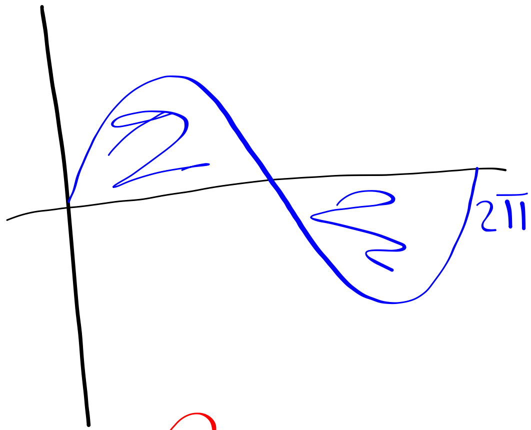
$$Area_2 = \left| \int_a^b f(x)\,dx \right| = -\int_a^b f(x)\,dx$$

2) Use geometry to evaluate integrals.

S.O.W. . . . .

$$\int_0^{2\pi} \sin x\,dx = ?$$

$$= 0$$



QUESTION $\int_0^{\pi} \sin x\,dx$?

5.4 Fundamental Theorem of Calculus

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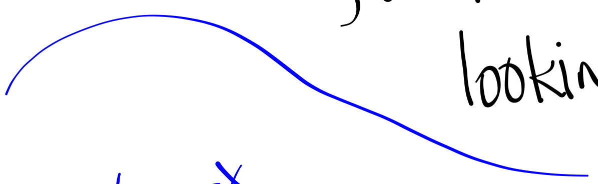
Part 1

semi definite integral

$\int_a^x f(t)\,dt$ is actually the area function Newton & cronies were looking for [Isaac Barrow]



$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

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Part 2 If $f(x)$ is integrable on $[a,b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is ANY antiderivative

$$\int_0^{\pi} \sin x\,dx = \left(-\cos x \boxed{+C}\right)\Big|_0^{\pi}$$

$$\int \sin x\,dx = -\cos x + C$$

$$= \left(-\cos \pi\right) + C - \left(-\cos(0) + C\right)$$

$$= -(-1) + C - (-1) - C$$

$$= +1 + C + 1 - C$$

$$= 2$$

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5.2/9) $v(t)=3t^2+1$ on $[0,4]$

a) $n=4$, so $[0,1],[1,2],[2,3],[3,4]$

midpt sum

$=(1)\cdot f(.5)+(1)\cdot f(1.5)+(1)\cdot f(2.5)+(1)\, f(3.5)$

$=3\left(\frac{1}{2}\right)^2+1+3\left(\frac{3}{2}\right)^2+1+$

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5.2/43

$n=40$;

$0 \quad 0+\frac{1}{10} \quad 0+2\left(\frac{1}{10}\right) \quad 0+3\left(\frac{1}{10}\right) \quad \ldots \quad 4$

$\frac{4}{40}=\frac{1}{10}$ $\qquad 0+40\frac{1}{10}$

LHS

$$\sum_{k=1}^{40}\left(\frac{1}{10}\right)\left(\sqrt{0+\frac{k-1}{10}}\right)$$

width height

$$=\sum_{k=0}^{39}\frac{1}{10}\cdot\sqrt{0+\frac{k}{10}}$$

$\sqrt{1}+\sqrt{2}+\sqrt{3}+\ldots$

DOES NOT EQUAL

$\sqrt{1+2+3+\ldots}$

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CALC I

2

3

Diff Eq's

MATH

REAL ANALYSIS

$\exists$ MULTIPLE THEORIES of Integration

$$\int_1^\infty \frac{1}{x}\,dx$$

vs

- Riemann Integration
- Lebesgue integration

Bon discontinuities